Calculus and Biology

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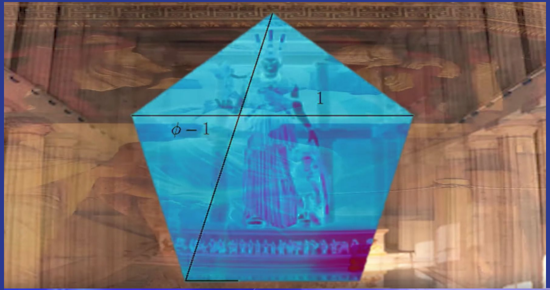
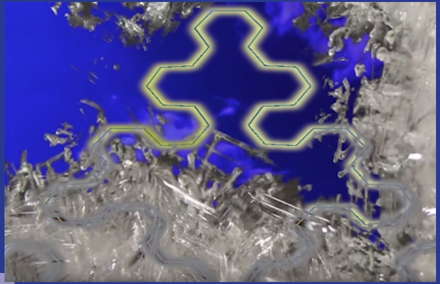
Background

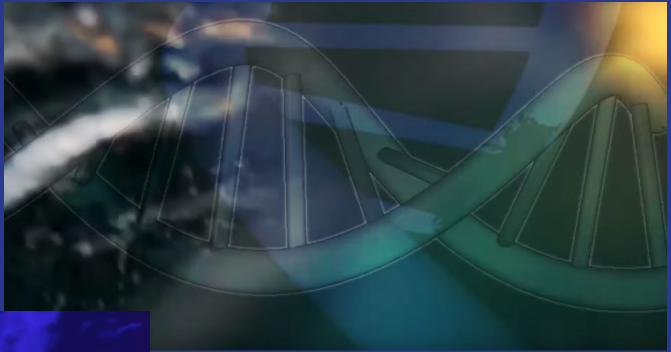
Last year, I worked on a project that compared the mathematical relationship between sine and cosine waves, from a historical standpoint, and how it arises in nature.

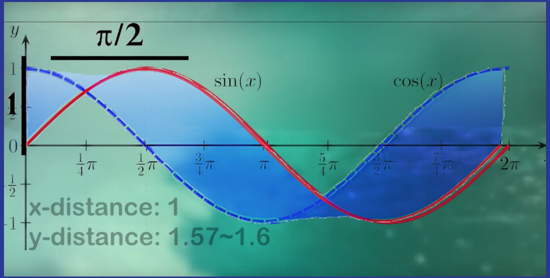
It was a Nova-documentary style video that mentioned Pythagorean, Euclid, DaVinci, Fibonacci, Luca Pacioli demonstrating the correlation between the golden ratio and its occurrences in biology, such as DNA, physics, such as behavior of electrons, and ecology, such as the structure of plants and the behavior of water.

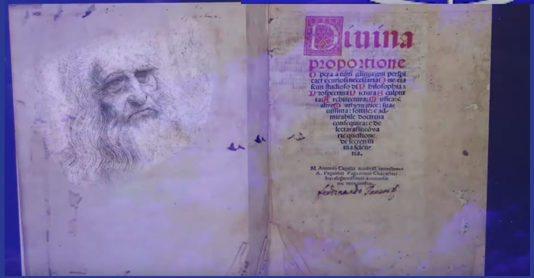
A Square Unit Of Space and Time

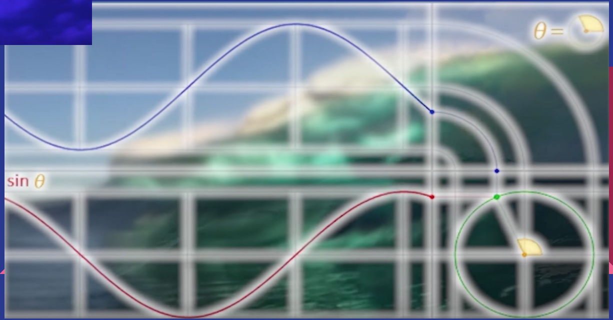












How this is related to calculus

My goal is to extend the same line of thinking in order to integrate what I have learned recently in calculus.

Differential equations can characterize the dynamics of a system as time approaches infinity. The behavior, illustrated as vectors that describe the rate of a population's growth, oscillating around points of equilibriums, brings forth similar patterns in nature to those previously mentioned.

How this is related to calculus

A more cohesive and comprehensive look at simple ecological relationships in terms of their species populations over time uses systems of differential equations to formulate predictions based on the stability of a systems equilibrium points.

The models for such systems often hold resemblance to Fibonacci numbers, and other such oscillating trend patterns.

Lotka-Volterra differential equations

The Lotka-Volterra equations are a pair of first-order, non linear, differential equations, frequently used to describe the dynamics of biological systems in which two species interact, one as a predator and the other as prey.

X= prey species (rabbits)

Y= predator species (foxes)

Lotka-Volterra differential equations

Lotka-Volterra systems model predator / prey relationships.

The two population depend on each other according to the following equations:

Rate of x population --- $x' = \alpha x - \beta xy$

Rate of y population --- $y' = -\delta x + \gamma xy$

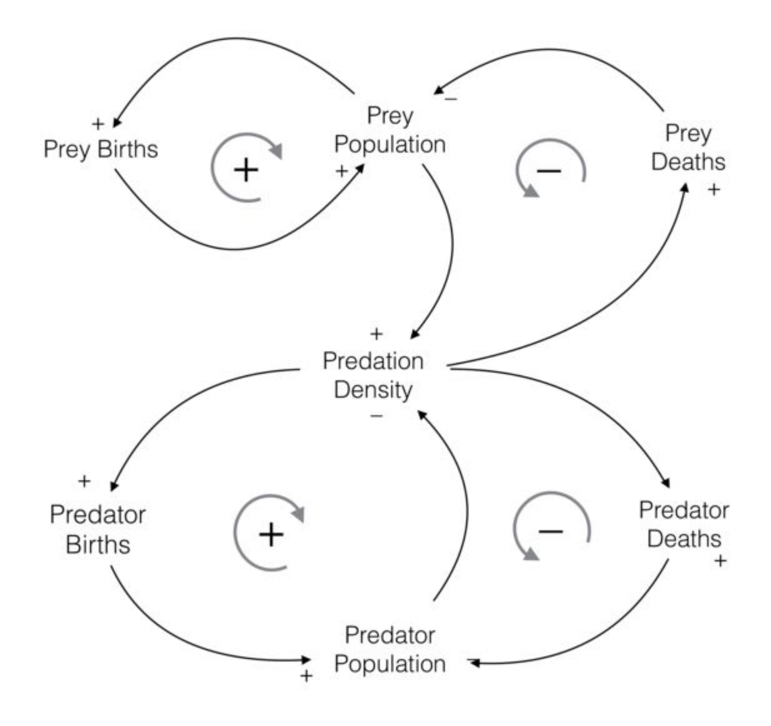
α net growth of rabbits

β fraction of interaction between rabbits and foxes that results in rabbit being eaten

δ net growth of foxes

γ degree to which an interaction increase the fox population.

GRAPHS

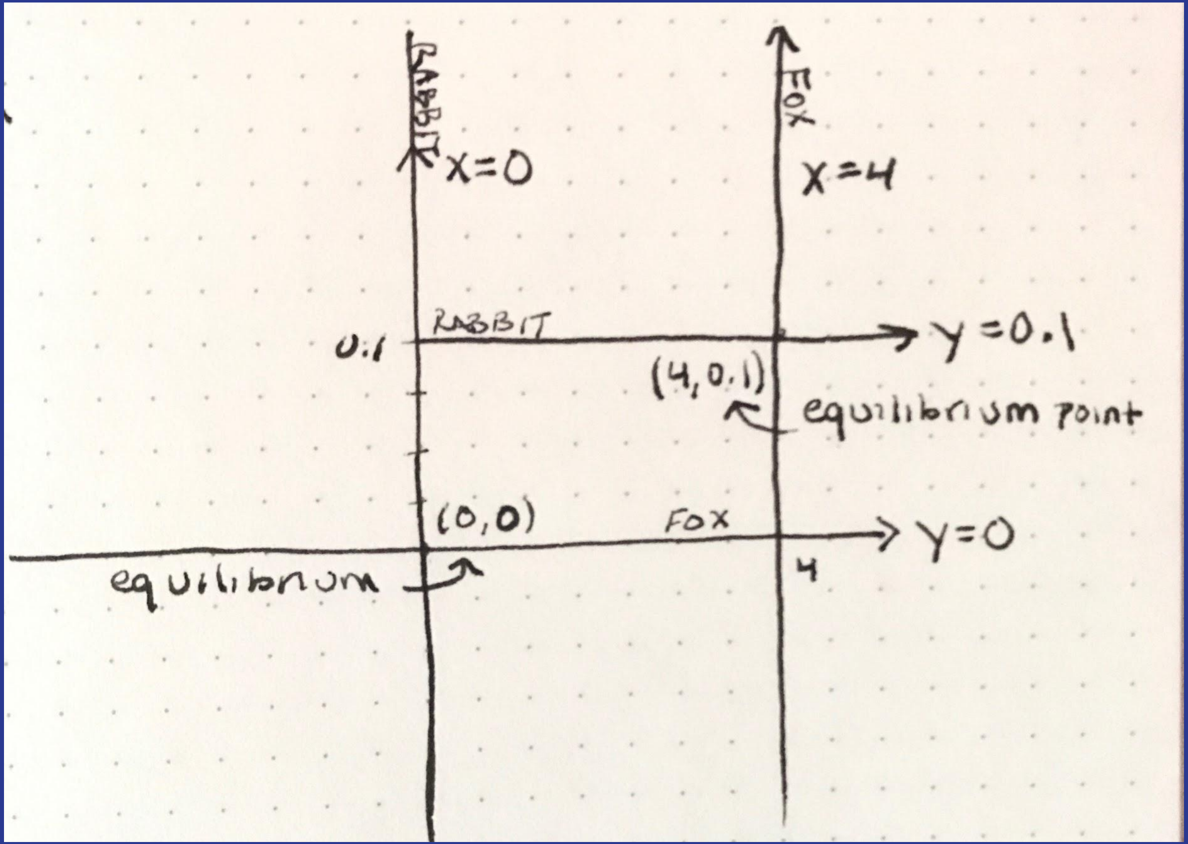


Many difference and differential equations are assisted by these input output models preliminarily

EQUILIBRIUM

To find the equilibrium points, or the points at which the population is growing at a rate of 0, set the equations to 0 since they are the derivatives of the populations x and y, and solve.

The result should provide 2 equations for lines as well as the x and y axis. Where x lines and y lines intersect are the points of equilibrium.

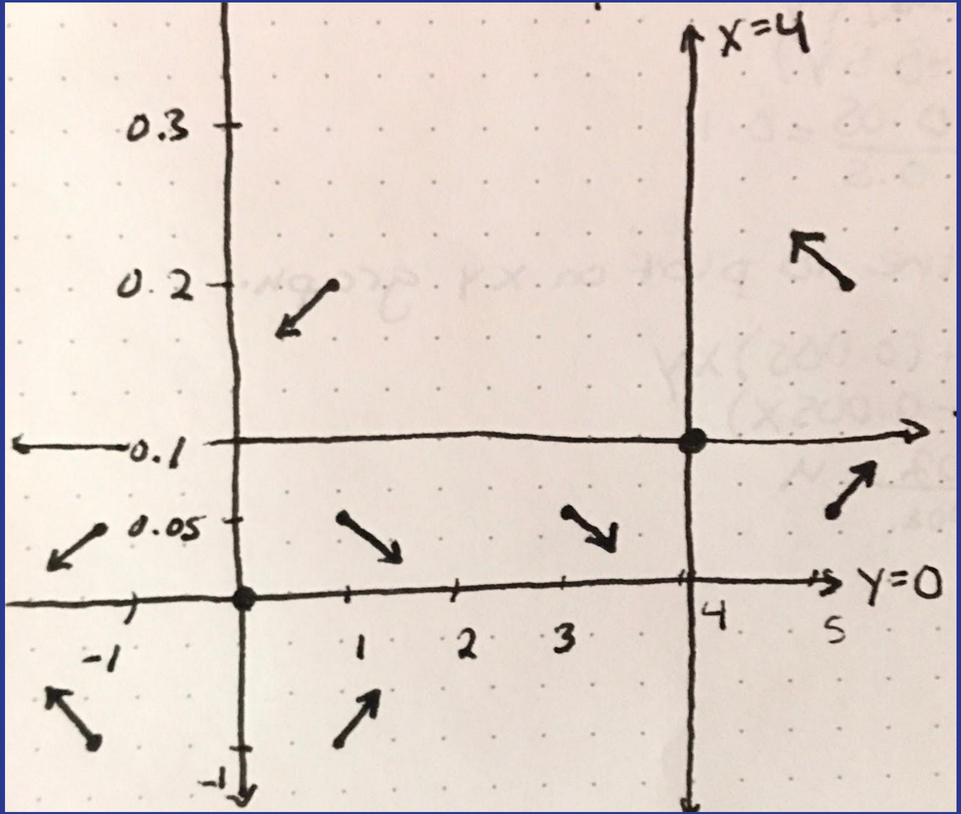


RABBIT
FOX
X=0
X=4
RABBIT
0.1
Y=0.1
(4,0.1)
equilibrium point
(0,0)
FOX
Y=0
4
equilibrium

Stability

A point of equilibrium within a dynamic system is stable when the rate of growth at various arbitrary coordinates surrounding or near the equilibrium, tend towards convergence with that equilibrium point.

One way to visualize stability is by plotting those arbitrary coordinates in relationship to the equilibrium points in the form of a vector based on the sign of value of the function given each set of coordinates.



(X,Y)	X'	Y'	X	Y
(1,0.05)	+	-	Increasing	Decreasing
(-1,0.05)	-	-	Decreasing	Decreasing
(-1,-1)	-	+	Decreasing	Increasing
(1,-1)	+	+	Increasing	Increasing
(5,0.05)	+	+	Increasing	Increasing
(5,0.2)	-	+	Decreasing	Increasing
(3,0.05)	+	-	Increasing	Decreasing
(1,0.2)	-	-	Decreasing	Decreasing

STABILITY

Another way to determine stability is called linearization in which you find the eigenvalues λ at each equilibrium point. The sign of the dominant eigenvalue determines stability.

Positive means unstable

Negative means stable

Complex eigenvalues

However, sometimes these values can be complex values.

It is when the eigenvalues are complex or imaginary that the natural spiral pattern occurs in the phase plot.

Complex values, when given in the form $\lambda=a+bi$,

If $a<0$, equilibrium is stable, oscillations decrease in size continuously over time, whereas if $a>0$, it is unstable because the oscillations gravitate further from the center.

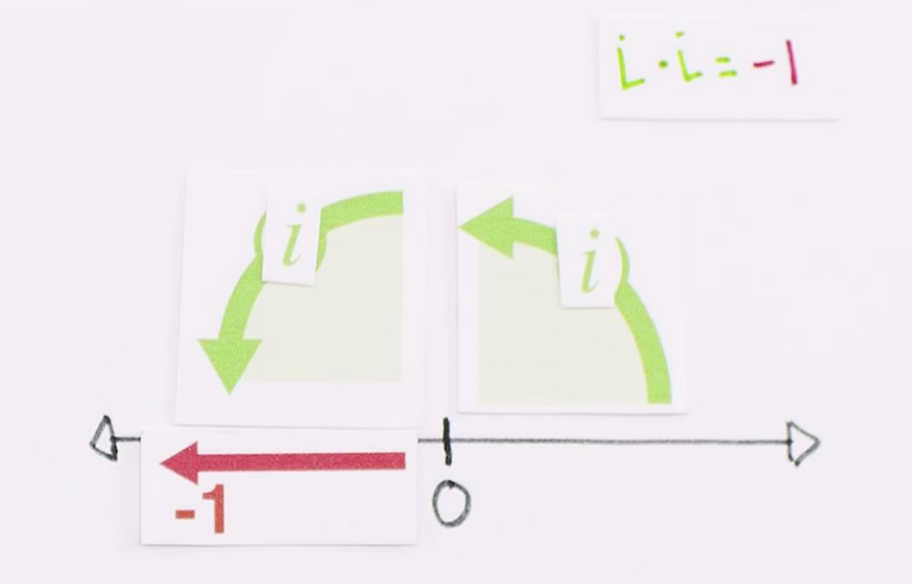
If $a=0$, on the other hand, oscillations remain the same.

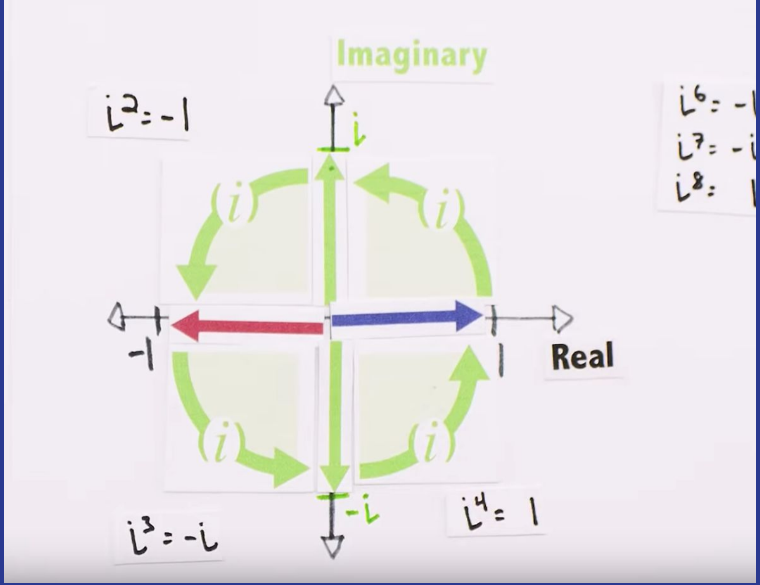
Complex eigenvalues

Complex values inherently lead to the oscillating pattern due to their rotational nature, in which they are defined by their magnitude or distance from the origin and their angle with respect to the real axis.

Why i can be the square root of -1 is because while a flip from a positive and negative value can be considered 180 degrees, the transition by multiplying by i, is only 90 degrees.

VISUALIZATION

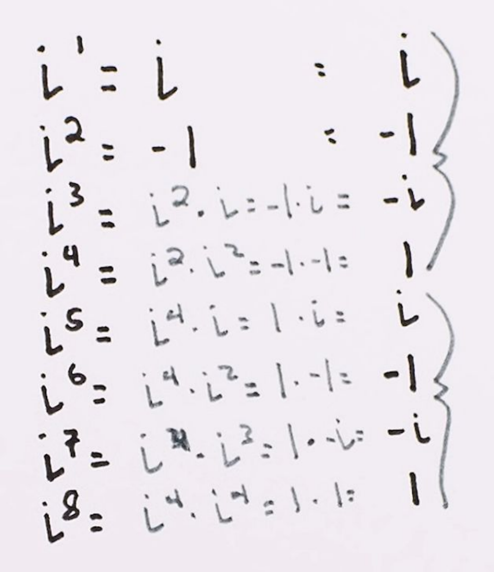




I raised to higher power

When imaginary number, i, is raised to greater and greater exponents, it doesn't increase in value continuously, it forms a pattern. These 4 values will repeat over and over again.

This pattern represents the same magnitude being rotated 90 degrees 4 times to end up back at one, only some time later. It is emulated in the linearized model depicting the population of predators trailing that of prey by 90° in the cycle and also in the 4 steps listed followed by "cycle repeats" in the predator/prey example to follow.



Graphs

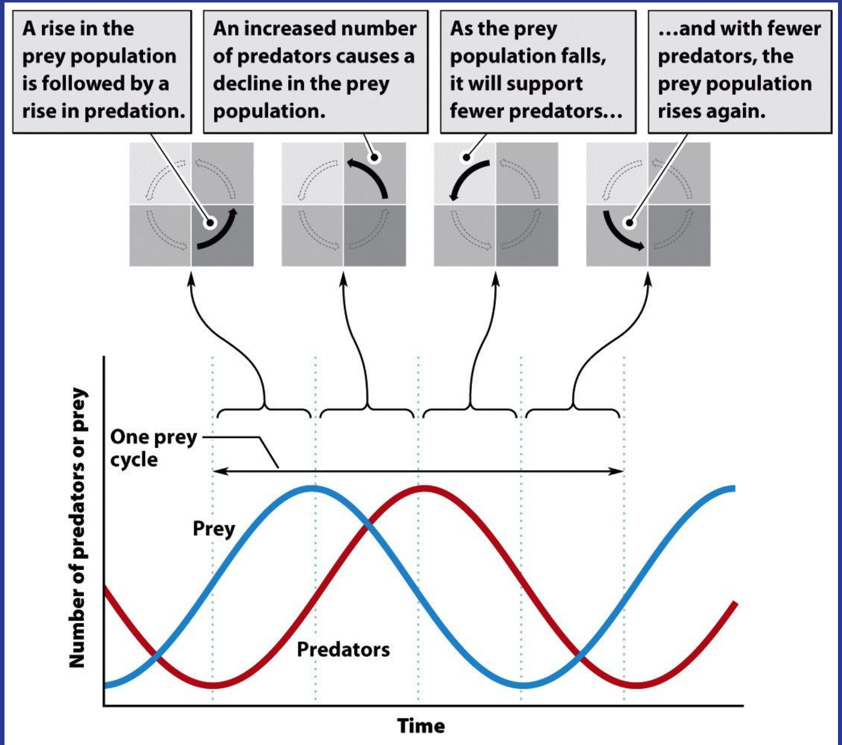
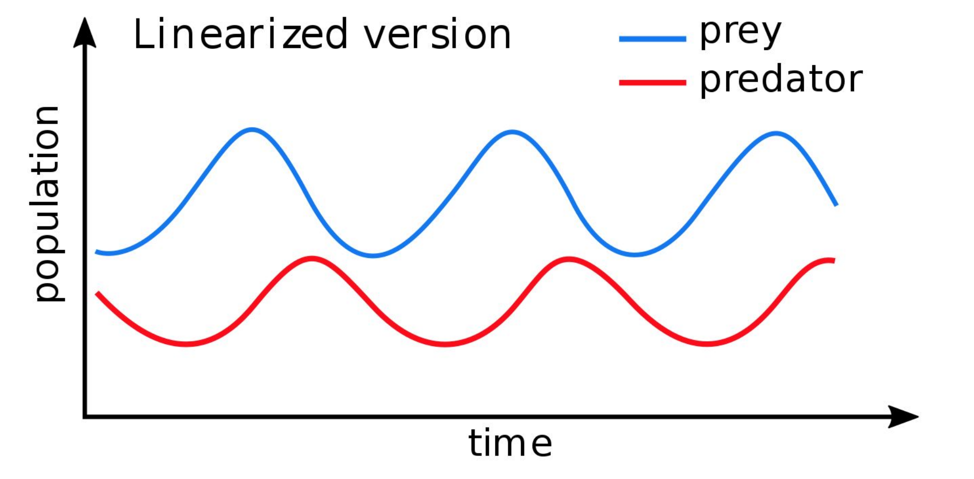
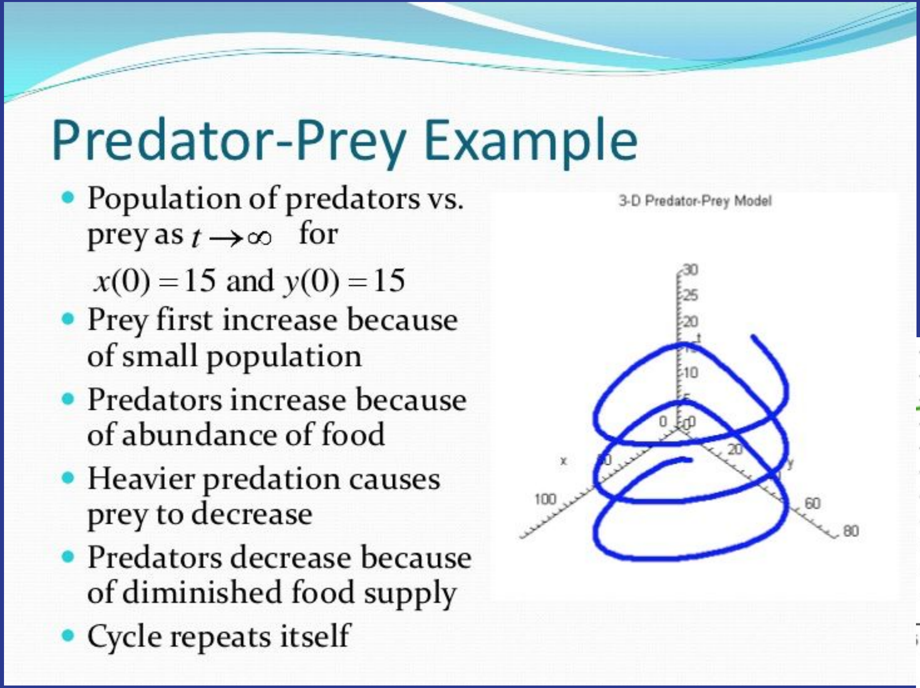


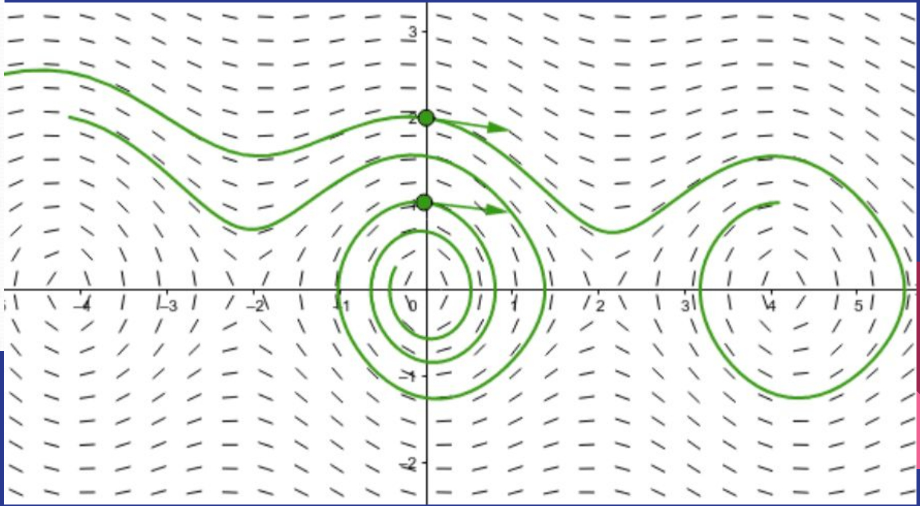
Figure 15.15
The Economy of Nature, Sixth Edition



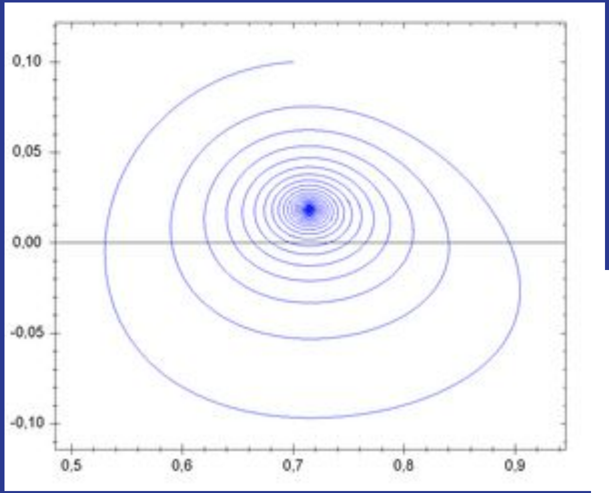
Graphs

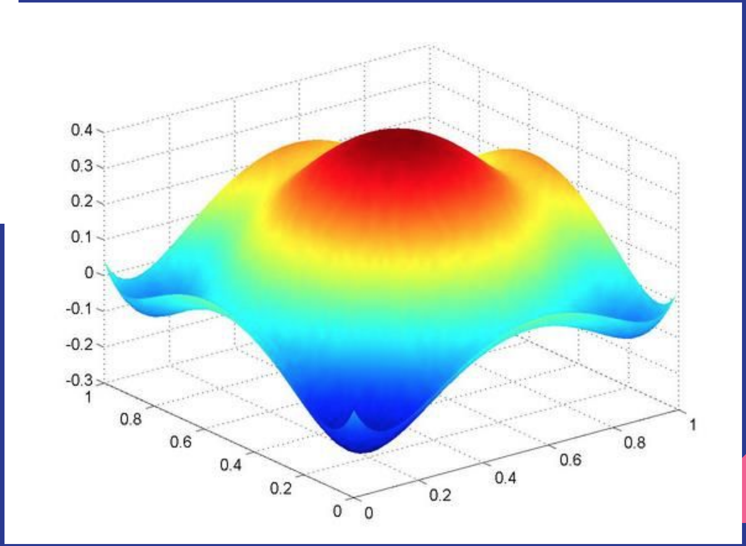


3D graphs as well as phase plots can model relationship at all times.



Graphs





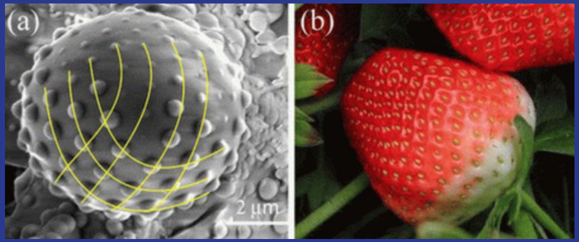
There are other ways to visualize these dynamic systems

Fibonacci

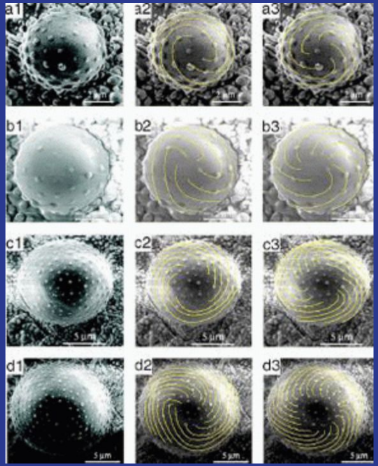
Fibonacci, in the first book to mention the Fibonacci sequence, Liber Abaci (1202) written by himself, uses hypothetical rabbit populations with input and output values over a discrete period of time.

There are also many other instances of its applicability.

Other applications in biology



"Scientists Find Clues to the Formation of Fibonacci Spirals in Nature"

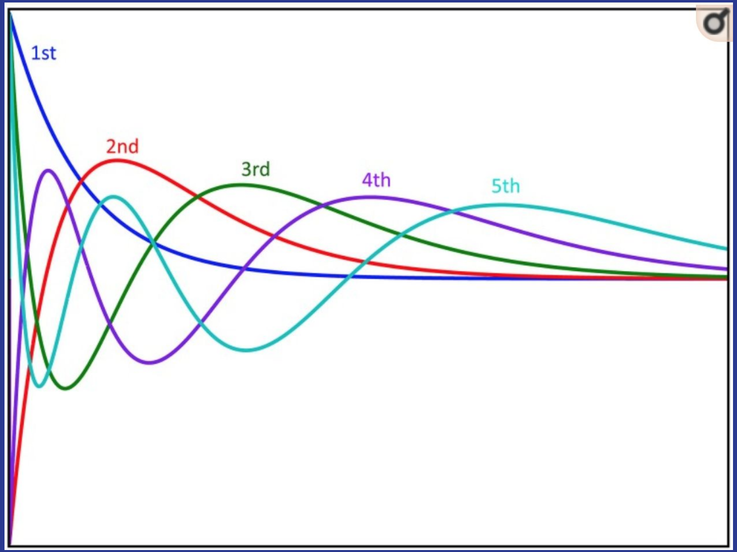


"Patterns that evolve naturally are generally an optimized configuration for an assembly of elements under an interaction," Cao explained to *PhysOrg.com*. "We conjecture that the Fibonacci spirals are the configuration of least elastic energy. Our experimental results provide a vivid demonstration of this energy principle."

"Chaorong Li, of the Zhejiang Sci-Tech University and the Institute of Physics in Beijing, along with Ailing Ji and Zexian Cao, both of the Chinese Academy of Sciences, produced their Fibonacci spiral pattern by manipulating the stress on inorganic microstructures made of a silver core and a silicon dioxide shell."

Other applications in biology

"Volterra representation enables modeling of complex synaptic nonlinear dynamics in large-scale simulations"



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sources

https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4585022/

https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4585022/

Welch Labs "Imaginary Numbers are Real: (Parts 1-9)

https://www.youtube.com/watch?v=65wYmy8Pf-Y&list=PLiaHhY2iBX9g6KIvZ_703G3KJXapKkNaF&index=5

https://en.wikipedia.org/wiki/Lotka%E2%80%93Volterra_equations